to say, it is a program limitation rather than a limitation in the method, and may be removed, if desired, by using quadrature formulas of higher precision.

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Analysis of the Power Loss in the Coupling Mechanism of a Cavity Resonator

M. DANIELSEN

***Abstract*—An analysis of a lossy coupling mechanism is presented. It is found that the reciprocal internal Q factor is augmented by a contribution $(1/2)\zeta(1/Q_{ex})$, where ζ depends on the parameters of the coupling mechanism and Q_{ex} is the external Q factor. The theory is applied to analyze the coupling to a superconducting high-Q cavity.**

Measurements on superconducting cavities with very high Q values have shown that coupling through a lossy coupling mechanism offers a special problem for obtaining the highest theoretical Q. Sucher and Fox [1] and Halbritter *et al.* [2] reported a decrease in the measured internal quality factor Q_0 of a cavity due to a lossy coupling mechanism. In the work on superconducting cavities, the author has analyzed the influence of the losses in a lossy coupling mechanism on the measured Q_0 of the cavity.

The analysis is performed as an analysis of the measurement of the internal Q_0 of the equivalent GCL circuit of the cavity through the equivalent lossy two-port of the coupling mechanism (Fig. 1).

The coupling factor at the input to the cavity is defined by

$$r = \frac{Q_0}{Q_{ex}} \tag{1}$$

where Q_{ex} is the external Q of the cavity seen from the two-port. The reflection coefficient at the input of the cavity, II, is then given by

$$\frac{a_2}{b_2} = \Gamma = \frac{r - 1 - jx}{r + 1 + jx} \tag{2}$$

where

$$x = \frac{2\Delta\omega}{\omega_0} Q_0 \tag{3}$$

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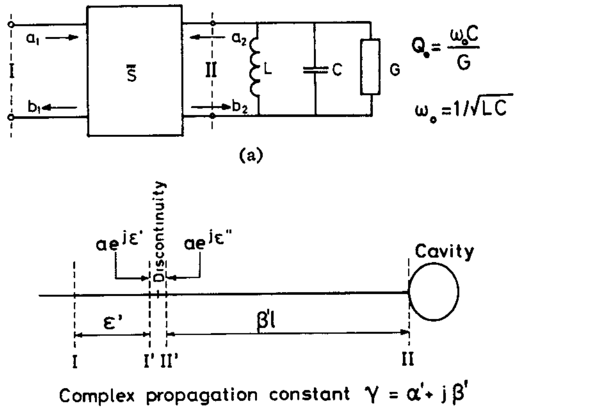

(b)

Fig. 1. (a) Equivalent circuit of cavity and coupling two-port. (b) Cavity with transmission-line coupling two-port.

and

$$\Delta\omega = \omega - \omega_0 \tag{4}$$

is the deviation of the frequency from the resonant frequency.

The scattering matrix of the two-port is

$$\bar{S} = \begin{Bmatrix} a & j\sqrt{b}e^{j\theta} \\ j\sqrt{b}e^{j\theta} & ce^{j\phi} \end{Bmatrix} \tag{5}$$

where a is real, if the reference plane at the input is selected suitably. The determinant is given by

$$\det \bar{S} = \Delta e^{j\eta} = ace^{j\phi} + be^{2j\theta} \tag{6}$$

where Δ is the magnitude and η the phase of the determinant.

The reflection coefficient squared at the input of the two-port, I, is found to be

$$|\Gamma'|^2 = |\Gamma_\infty'|^2 \frac{(x-\alpha)^2 + A^2}{(x-\beta)^2 + B^2} \tag{7}$$

where

$$|\Gamma_\infty'|^2 = \frac{\Delta^2 + 2\Delta a \cos\eta + a^2}{1 + 2c\cos\phi + c^2} \tag{8}$$

$$\alpha = \frac{2\Delta a \sin\eta}{\Delta^2 + 2\Delta a \cos\eta + a^2} r \tag{9}$$

$$\beta = \frac{2c\sin\phi}{1 + 2c\cos\phi + c^2} r \tag{10}$$

$$A = \left| r \frac{\Delta^2 - a^2}{\Delta^2 + 2a\Delta\cos\eta + a^2} - 1 \right| \tag{11}$$

$$B = \left| r \frac{1 - c^2}{1 + 2c\cos\phi + c^2} + 1 \right|. \tag{12}$$

It is seen that when $\alpha \neq \beta$, the resonance is unsymmetric and can have both a maximum and a minimum. In most conventional cavities the unsymmetry is not seen because the phase shift through the two-port is negligible, i.e., $\phi = \eta = 0$. It should be possible to determine all the involved parameters by measuring $|\Gamma'|^2$ as a function of the frequency. However, for high-Q cavities it is normally difficult to measure the response with sufficient accuracy because of the difficulty in accurately determining the frequency. Further, from an application point of view, e.g., application to narrow-band filters and frequency stabilization, it is desirable to obtain a resonance curve with a small bandwidth. Therefore, the subsequent treatment is concentrated on the derivation of the internal Q expressed by the measured internal Q at the input of the two-port.

It is assumed that the two-port parameters are frequency independent over the bandwidth of the resonator. $|\Gamma'|^2$ has a minimum value $|\Gamma'|_m^2$ at resonance, $x = x_m$. In the following derivations it is assumed that $|x_m - \beta| \ll B$ and $A < B$. It is then found that

$$x_m = \frac{B^2\alpha - A^2\beta}{B^2 - A^2} \tag{13}$$

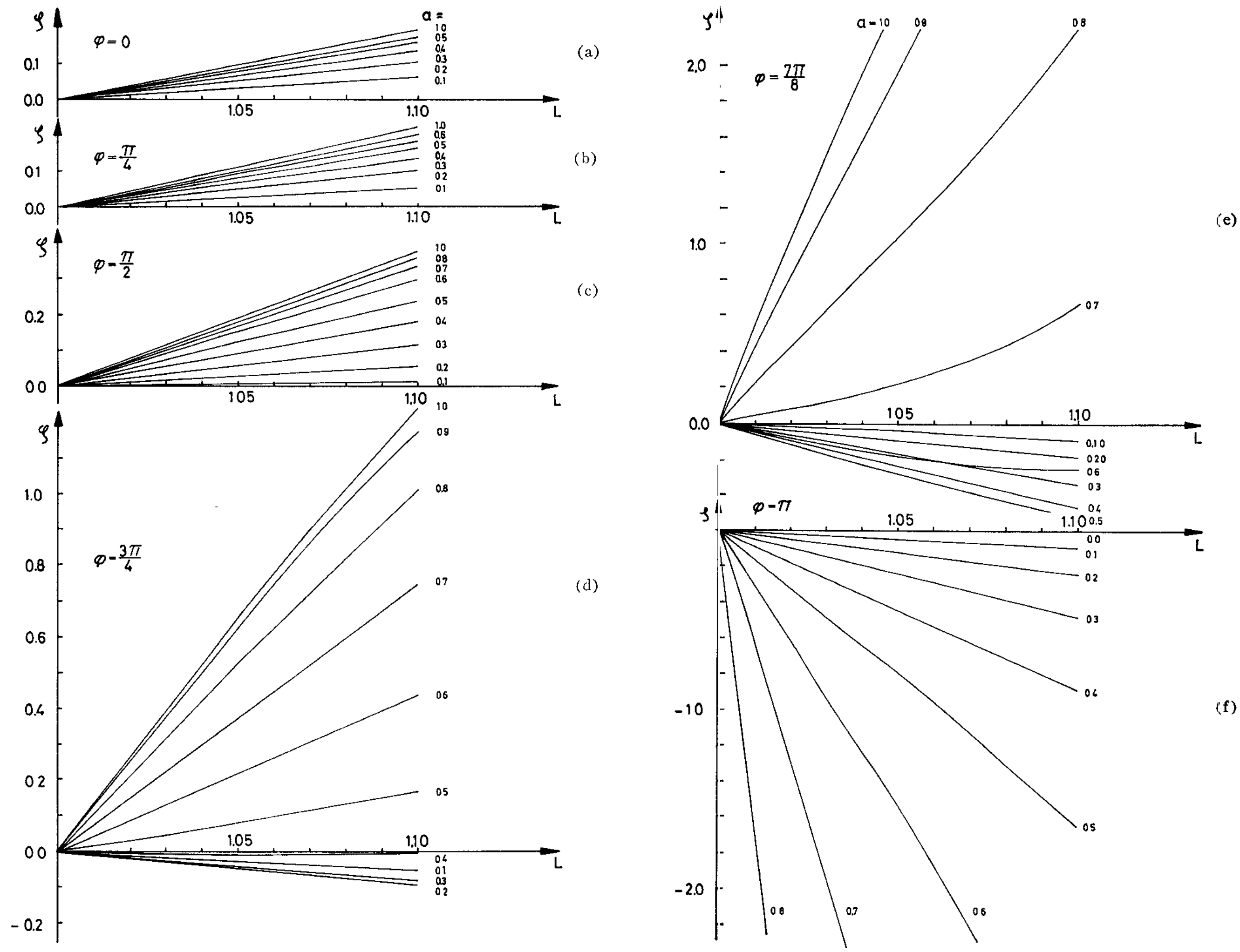

Fig. 2. Correction parameter ζ for internal Q as a function of attenuation in transmission line L, with phase ϕ of S_{22} and $a = |S_{11}|$ as parameters.

and

$$|\Gamma'|_m^2 = \frac{A^2}{B^2}|\Gamma_\infty'|^2. \tag{14}$$

The half-power points x_1 and x_2, defined as the values where $|\Gamma'|^2 = \frac{1}{2}(|\Gamma_\infty'|^2 + |\Gamma'|_m^2)$, are found to be

$$\left.\begin{matrix} x_1 - \alpha \\ x_2 - \alpha \end{matrix}\right\} = \frac{A^2+B^2}{A^2-B^2}(\alpha-\beta) \pm \sqrt{\left[\frac{A^2+B^2}{A^2-B^2}\right]^2(\alpha-\beta)^2 + B^2} \tag{15}$$

and the half difference between x_1 and x_2 corresponding to the full half-power frequency width $2\Delta\omega_{1/2}$ (which actually is measured) is then

$$\Delta x_{1/2} = \tfrac{1}{2}|x_1 - x_2| = \sqrt{\left|\frac{A^2+B^2}{A^2-B^2}\right|^2(\alpha-\beta)^2 + B^2} \approx B$$

when

$$|\alpha - \beta| \ll \frac{B(A^2-B^2)}{A^2+B^2} \approx \frac{2r(r+1)}{r^2+1}. \tag{16}$$

The last condition is always fulfilled for small losses. B can be expressed by the measurable quantities $|\Gamma_\infty'|^2$ and $|\Gamma'|_m^2$ and a correction factor $(B \pm A)/2$. The internal Q_0 is then found to be

$$Q_0 = \frac{\omega_0}{2\Delta\omega_{1/2}}\cdot\Delta x_{1/2} = \frac{2}{1 \pm \dfrac{|\Gamma'|_m}{|\Gamma_\infty'|}}Q_L\frac{B \pm A}{2} \tag{17}$$

where $Q_L = \omega_0/2\Delta\omega_{1/2}$ is the loaded Q of the cavity. The sign + is used when $r<1$ and the sign − when $r>1$. The measured internal Q is given by

$$Q_{0,\text{measured}} = \frac{2}{1 \pm \dfrac{|\Gamma'|_m}{|\Gamma_\infty'|}}Q_L. \tag{18}$$

The correct value of the internal Q_0 is then given by the insertion of (11) and (12) into (17):

$$Q_0 = Q_{0,\text{measured}}(1 + \tfrac{1}{2}\zeta r) \tag{19}$$

where the correction parameter is given by

$$\zeta = \frac{1-c^2}{1+2c\cos\phi + c^2} - \frac{\Delta^2 - a^2}{\Delta^2 + 2a\Delta\cos\eta + a^2}. \tag{20}$$

From (20) it is seen that if the two-port is lossless, i.e., $\Delta = 1$, $\phi = \eta$, and $a = c$, then the correction parameter $\zeta = 0$, and $Q_{0,\text{measured}} = Q_0$. Further, it follows that if the two-port is matched, i.e., $a = c = 0$, then $\zeta = 0$. If these conditions cannot be obtained, a nearly correct measured value of the internal Q_0 can be obtained by setting $r \ll 1$, i.e., loose coupling.

The theory is important for all coupling mechanisms, as there must always be a transducer between the cavity and the transmission line, where electromagnetic fields that cause some losses are present.

In the work on superconducting cavities [3] the author has analyzed a coupling mechanism [Fig. 1(b)] consisting of a lossy transmission line of length l, the complex propagation constant $\gamma = \alpha' + j\beta'$, and a reflecting discontinuity with the complex reflection coefficient $ae^{j\phi'}$ from the left and $ae^{j\phi''}$ from the right (a is real). Now the scattering matrix of the two-port with the reference planes I' and II' is

$$\bar{S}' = \begin{Bmatrix} ae^{j\epsilon'} & j\sqrt{1-a^2}\,e^{(j/2)(\epsilon''+\epsilon')} \\ j\sqrt{1-a^2}\,e^{(j/2)(\epsilon''+\epsilon')} & ae^{j\epsilon''} \end{Bmatrix}. \tag{21}$$

If the reference planes are shifted to I and II, the scattering matrix becomes

$$\bar{S} = \begin{Bmatrix} a & j\sqrt{1-a^2}\,e^{j[(\epsilon''/2)-\beta'l]}L^{-1} \\ j\sqrt{1-a^2}\,e^{j[(\epsilon''/2)-\beta'l]}L^{-1} & ae^{j(\epsilon''-2\beta'l)}L^{-2} \end{Bmatrix} \tag{22}$$

where $L=e^{\alpha'\cdot l}$ is the attenuation of the transmission line.

From (22) the relevant parameters can be deduced:

$$\begin{aligned} \eta &= \phi = \epsilon'' - 2\beta' l \\ \Delta &= L^{-2} \\ c &= aL^{-2} \end{aligned} \tag{23}$$

and the correction parameter is found to be

$$\zeta = \frac{L^4 - a^2}{L^4 + 2L^2 a\cos\phi + a^2} - \frac{1 - L^4a^2}{1 + 2L^2a\cos\phi + L^4a^2}. \tag{24}$$

Fig. 2 shows a plot of ζ. The negative value of ζ for ϕ values near π can be understood qualitatively from the fact that the cavity can have a small coupling factor r, but the apparent coupling factor measured at the input of the two-port is larger. The bandwidth is determined by the cavity and the small coupling factor. Calculation of Q_0 from the bandwidth and the large coupling factor therefore gives too high a value of Q.

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A Microstripline Slot Antenna

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***Abstract*—A microstripline slot antenna is treated experimentally at X-band frequency. First, the input impedances of the slots for various geometries and the radiation patterns for the matched slots are measured. Second, the dependence of input impedances and radiation patterns on the slot-to-reflector spacing is tested. Finally, a two-dimensional X-band Dolph–Chebyshev slot-array antenna is designed and fabricated as an application of this type of slot.**

I. Introduction

Microstripline slot antennas appear to be quite useful in various fields of application because of their savings in cost, size, and weight. Concerning an antenna utilizing microstriplines, printed antennas, such that several types of array patterns are printed on the substrates of microstriplines, have been reported by several authors [1], [2], and slot antennas utilizing triplate striplines have also been reported by Oliner [3], Sommers [4], and Breithaupt [5]. Slot antennas utilizing microstriplines have also been investigated by the author [6].

II. Slot Configuration

A microstripline slot antenna shown in Fig. 1 is fabricated by simple and conventional photoetching techniques from copper-clad PTFE Fiberglas laminate. The width of the strip conductor is so determined that the characteristic impedance of the microstripline is 50 Ω. The slot is so made that its longer sides are perpendicular to the strip conductor. The method of the feeding is as follows: the strip conductor is short-circuited through the dielectric substrate with the slot longer side, which is located farther from the feed input. No matching element is used because such an element decreases the bandwidth. It can be predicted that the radiation patterns of the slot are disturbed, with a certain amount due to the influence of the strip conductor near the feed point and the dielectric substrate on one side of the slot surface. The feed point on the slot can be divided into two cases; one, the feed point dislocated from the center of the longer side (which is called the offset-fed slot), and two, the feed point located on the center (which is called the center-fed slot).

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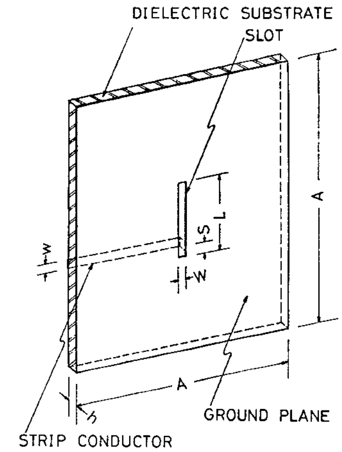

Fig. 1. Configuration of microstripline slot antenna.

III. Input Impedances and Radiation Patterns

The microstripline slot antenna is measured after transforming a 50-Ω microstripline to a 50-Ω coaxial line. The thickness of the dielectric substrate, the width of the strip conductor, and the size of the ground plane are 0.6 mm, 1.55 mm, and 64 mm by 64 mm, respectively, and the relative dielectric constant of the dielectric substrate is nominally 2.7 at the frequency of 10 GHz. The measurement frequency is 10.525 GHz, which corresponds to the free-space wavelength of 28.5 mm.

The impedances of the slots are measured as functions of the lengths, widths, and feed points of the slots. Fig. 2(a) illustrates the measured data of input impedances of the offset-fed slots as functions of the lengths and the feed points with the slot widths kept constant. The input impedances indicate the impedances at the feed point where the strip turns 90° to go through the substrate. It is found that in the case of the offset-fed slot, the slot with a size of $L=13.5$ mm, $W=0.7$ mm, and $S=3.5$ mm is matched to the feed line of the 50-Ω microstripline. Measurement data for the center-fed slot are shown in Fig. 2(b). It is found from the comparison of the sizes of the matched slots for both cases that the length from the feed point to the farther edge for the offset-fed slot is almost the same as the length from the feed point to one edge for the center-fed slot. From this fact, it could be predicted that the field distributions for these two parts would be similar [6].

The measurement results of the radiation patterns of the matched slots are shown in Fig. 3 for the offset-fed slot. The H-plane radiation patterns for both cases are quite similar to the E-plane patterns for dipole antennas, and no distortions of the H-plane patterns can be noticed. The beamwidth in the H plane is about 68° for the offset-fed case. Concerning the E-plane pattern, the wave-like radiation pattern illustrated in the figure is considered to have come from the interference of the radiations from the slot and the edges of the ground planes [7]. If an infinite ground plane could be used in essence, the E-plane radiation pattern should be uniform. Thus this type of slot can be regarded as an antenna complementary to a dipole antenna, except for the effect of the finiteness of the ground plane. The distortions of the radiation patterns due to the strip conductor and the dielectric substrate are hardly noticed contrary to the prediction previously stated. This can be explained by the fact that the effective thickness of the dielectric substrate is very small compared with the free-space wavelength; therefore, the far field is hardly influenced by the existence of the dielectric substrate. Furthermore, since the width